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Resonator induced plasmon filter: theoretical study

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Abstract

We study theoretically the plasmon mediated propagation of light through a chain of metallic nanoparticles along which a small chain (called a resonator) is attached vertically. The effect of this vertical resonator is to induce peaks and zeros in the transmission power. We show that, with a resonator constituted of two metallic clusters, an appropriate choice of the geometrical parameters can lead to a narrow peak in the transmission spectrum. This may be realized for this device by adjusting the distances between the two resonator clusters and the chain cluster to which they are attached. This enables us to get, in particular, a sharp peak between two zeros of transmission close to each other or sharp dips between two transmission ones. Such a device can be useful as a selecting or rejecting plasmon filter.

In recent years, significant progress has been made towards reducing the size of optical devices. This trend towards miniaturization is driven by the increase in system functionality and reduction in power dissipation that may be achieved when highly integrated photonic networks replace today's discrete devices and stand-alone modules. Another important motivation is a vision of an architecture in which photonic circuits integrate seamlessly into large-scale electronic systems. This requires waveguides that bridge the gap in size between conventional micron-scale integrated photonics and nanoscale electronics. Additionally, nanostructured materials often possess strong nonlinear properties that can be exploited in the development of novel active devices, since the confinement of light to small volumes can lead to nonlinear optical effects even with modest input power.

In purely dielectric materials, the optical diffraction limit places a lower bound on the transverse dimension of waveguide modes at about $\lambda_0/2n$ —i.e. several hundreds of nanometres

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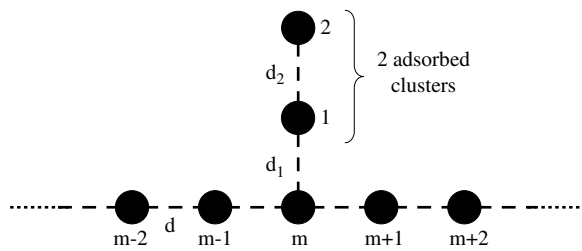


Figure 1. Sketch of the geometry of the nanometric device considered. It consists of one metallic cluster chain and two other clusters. The distances between these clusters are, respectively, d , d_1 and d_2 , as indicated in the figure. Each cluster is modeled by a point dipole characterized by an angular frequency ω_0 .

for visible light [1]. Plasmonic waveguides, on the other hand, employ the localization of electromagnetic fields near metal surfaces to confine and guide light in regions much smaller than the free-space wavelength, and can effectively overcome the diffraction limit.

In plasmonic systems there is generally a trade-off between the size of the electromagnetic mode and loss in the metallic structures. With this design principle in mind, there are several choices for plasmonic waveguiding technologies which may prove useful for various applications. For example, thin metal stripes support long-range surface plasmon polaritons with an attenuation length as long as millimetres, but lack subwavelength mode confinement [2, 3]. Another geometry is metallic nanowires, which indeed can provide lateral confinement of the mode below the optical diffraction limit. Nanowires have larger attenuation than planar films, but light transport over a distance of several microns has been demonstrated [4]. Finally, metal nanoparticles are used to achieve three-dimensional (3D) subwavelength confinement of optical-frequency electromagnetic fields in resonant ‘particle plasmon’ modes [5]. Nanoparticles provide highly enhanced local fields which are promising for molecular sensors [6, 7] or miniature nonlinear optical elements [8]. Due to the near-field interaction of surface plasmon–polariton modes of adjacent particles, an array of such particles can act as waveguides over modest distances [9]. Indeed, linear chains of metal nanoparticles have been shown to support coherent energy propagation over a distance of hundreds of nanometres [10, 11]. The minimum length scales in fabricated structures were determined by the resolution of electron-beam lithography, with particle diameters of $30 \times 30 \times 90 \text{ nm}^3$ and interparticle spacings of 50 nm [10].

In this paper, we consider such a linear chain of metallic nanoparticles and study the effect on the transmission spectrum of two additional clusters coupled to one single cluster of the chain, as depicted in figure 1. The effect of coupling the infinite wire to a local resonator is to induce peaks and dips (or zeros) in the transmission coefficient. The main purpose of this paper is to discuss the possibility of a narrow peak in the transmission by selecting appropriately the geometrical parameters of the problem. This enables us to get, in particular, a sharp peak between two zeros of transmission close to each other, for a simple two-lead device. Let us stress that such results were not obtained [12] for a similar two-lead device with continuous wires and resonators, for which only large dips were reported. Sharp peaks have been found before [12], but only for a more complicated four-lead device, namely a multiplexer. As shown in figure 1, we call d the distance between two clusters in the infinite chain, d_1 the distance between the adsorbed cluster molecule and the wire, and d_2 the distance between the two clusters in the resonator.

The problem is treated in the framework of a simple analytical model where each cluster is considered as a point dipole characterized by its angular frequency ω_0 . We describe the interactions between the clusters by a quasi-static approach, with retardation neglected. In this quasi-static limit, the Forster field ω_1^2 (used by Brongersma *et al* [13]) between two clusters is inversely proportional to the third power of the cluster separation, provided that the separation

is kept small compared to the wavelength λ . The radiation field, proportional to the inverse of the distance, dominates for large distances compared to the wavelength and is neglected here, as well as the losses due to the curvatures of the clusters and of the wires. The validity of the ω_1^2 dependence with cluster separation has been discussed by Maier *et al* [14] for greater values of the cluster separation compared to the wavelength, using a finite difference time domain (FDTD) calculation. In our simple model, we take into account the coupling only between the nearest-neighbour clusters. Of course, the above assumptions are the simplest possible approximations.

One can calculate the plasmon–polariton dispersion relation $\omega(k)$ for energy propagation along the nanoparticle chain waveguide by modelling the chain as a one-dimensional system of coupled damped harmonic dipole–dipole oscillators spaced a distance $d \ll \lambda$ apart. Each point-dipole m is attributed with the dipole moment p_m polarized perpendicular (transverse polarization) to the chain axis.

Taking the electromagnetic near-field between adjacent point-dipoles of transverse polarization into account, the equation governing the evolution of the dipoles in an infinite chain can be written as [13]

$$\frac{d^2 p_m}{dt^2} = -\omega_0^2 p_m - \Gamma_l \frac{dp_m}{dt} + \frac{\Gamma_R}{\omega_0^2} \frac{d^3 p_m}{dt^3} - \omega_1^2 (p_{m-1} + p_{m+1}). \quad (1)$$

This equation consists of four terms. The plasmon dipole resonance is described by a harmonic oscillator term at frequency ω_0 . The value of the frequency ω_0 depends on the geometrical parameters and the nature of the cluster [15]. The second term and third term model the damping of plasmon waves along the chain. The damping constant Γ_l is the electronic relaxation frequency due to interactions with phonons, electrons, lattice defects and impurities, and the damping constant Γ_R is the relaxation frequency due to radiation to the far field [16]. For Ag and Au nanoparticles in the point-dipole limits, ($\Gamma_R \ll \Gamma_l$) holds, so the radiating oscillator damping term can be neglected [13]. The fourth term incorporates the electrodynamic interaction with the nearest-neighbour dipoles at $m - 1$ and $m + 1$. This term is responsible for the existence of propagating solutions. The coupling strength is determined by the value of ω_1^2 , which is given by

$$\omega_1^2 = \frac{qe}{4\pi m^* \epsilon_0 n^2 d^3}, \quad (2)$$

where q is the magnitude of the oscillating charge, n is the refractive index of the host material, ϵ_0 is the free-space permittivity, m^* is the effective electron mass, and e is the electron charge.

Propagating wave solutions to equation (1) are of the form

$$p_m = p_0 \exp[i(\omega t - \tilde{k}md)], \quad (3)$$

where

$$\tilde{k} = k - i\alpha \quad (4)$$

is the complex wavevector and p_0 is the dipole moment at $m = 0$. The damping of the plasmon wave per unit length is given by the attenuation coefficient α . The angular frequency and the wavevector of the chain plasmon wave are given by ω and $k = 2\pi/\lambda$, respectively. Substituting equation (3) into equation (1), one obtains

$$\omega^2 - i\Gamma_l \omega - \omega_0^2 = \omega_1^2 (e^{i\tilde{k}d} + e^{-i\tilde{k}d}). \quad (5)$$

In order to give complex wavevector \tilde{k} as a function of ω , one can remark that equation (5) is a second-order equation when considering $e^{i\tilde{k}d}$ as the variable. Therefore, this last equation admits as solutions

$$e^{\pm i\tilde{k}d} = \frac{\omega^2 - \omega_0^2}{2\omega_1^2} - i\frac{\Gamma_l \omega}{2\omega_1^2} \pm \left[\left(\frac{\omega^2 - \omega_0^2}{2\omega_1^2} - i\frac{\Gamma_l \omega}{2\omega_1^2} \right)^2 - 1 \right]^{1/2}. \quad (6)$$

Equation (5) also provides two equations for the real and imaginary parts:

$$\omega^2(k) = \omega_0^2 + 2\omega_1^2 \cos(kd) \cosh(\alpha d) \quad (7)$$

and

$$\omega \Gamma_l + 2\omega_1^2 \sin(kd) \sinh(\alpha d) = 0. \quad (8)$$

Equation (7) is the dispersion relation for the plasmon–polariton waves. The element between cluster sites m and m' of the Green's function associated with such an infinite linear chain is well known to be [12]

$$G(m, m') = \frac{i e^{i\tilde{k}a|m-m'|}}{2\omega_1^2 \sin(\tilde{k}a)}. \quad (9)$$

One considers a one-dimensional system formed out of a finite chain grafted on an infinite plasmonic chain (see figure 1). In order to calculate the transmitted and reflected wavefunctions using the Green's function method [12], we construct this system with an infinite chain and a finite chain constructed of two clusters 1 and 2. These two blocks are coupled at their ends. The two clusters 1 and 2 are characterized by their angular frequencies ω_{01}^2 and ω_{02}^2 . For the infinite chain and for the finite chain, the interface domain correspond to sites m , 1 and 2, respectively.

The inverse Green's functions $G_1^{-1}(m, m)$ for the infinite chain and $g_2^{-1}(M, M)$, $M = [1, 2]$, for the grafted finite chain are given by:

$$G_1^{-1}(m, m) = -2i\omega_1^2 \sin(\tilde{k}a) \quad (10)$$

and

$$g_2^{-1}(M, M) = \begin{pmatrix} \omega^2 - \omega_{01}^2 & -\omega_{12}^2 \\ -\omega_{12}^2 & \omega^2 - \omega_{02}^2 \end{pmatrix} \quad (11)$$

where ω_{12}^2 represent the coupling term between the clusters 1 and 2.

Superposing these different contributions, one deduces [12] that the inverse interface Green's function of the composite system is

$$g^{-1}(M, M) = \begin{pmatrix} -2i\omega_1^2 \sin(\tilde{k}a) & -\omega_1^2 & 0 \\ -\omega_1^2 & \omega^2 - \omega_{01}^2 & -\omega_{12}^2 \\ 0 & -\omega_{12}^2 & \omega^2 - \omega_{02}^2 \end{pmatrix} \quad (12)$$

where $M = [m, 1, 2]$ and ω_1^2 represent the coupling term between the clusters m and 1.

In general, any incident wave, coming from $m = -\infty$, of amplitude unity launched onto the structure generates, as a result of scattering processes, the transmitted and reflected wavefunctions t and r which, with a Green's function method [12], are easily found to be

$$t = \frac{g(m, m)}{G(m, m)} = -2i\omega_1^2 \sin(\tilde{k}a) g(m, m) = \frac{1}{1 - iY_1}, \quad (13)$$

and

$$r = t - 1, \quad (14)$$

where

$$Y_1 = \left(\frac{d}{d_1}\right)^6 \frac{\cot(\tilde{k}d)}{4 \cos^2(\tilde{k}d) - (d/d_2)^6}. \quad (15)$$

Inserting equation (6) into (13), we arrive at the following equation of the transmission coefficient as a function of the frequency: ω

$$T = |t|^2 = \left| \frac{1}{1 - i \left(\frac{d}{d_1}\right)^6 \frac{Y_2}{\sqrt{1 - Y_2^2 [4Y_2^2 - (d/d_2)^6]}}} \right|^2 \quad (16)$$

where

$$Y_2 = \frac{\omega^2 - \omega_0^2}{2\omega_1^2} - i \frac{\Gamma_1 \omega}{2\omega_1^2} \quad (17)$$

and the reflection coefficient is

$$R = \left| 1 - \frac{1}{1 - i \left(\frac{d}{d_1}\right)^6 \frac{Y_2}{\sqrt{1 - Y_2^2 [4Y_2^2 - (d/d_2)^6]}}} \right|^2. \quad (18)$$

Let us remind ourselves that these coefficients are functions of the frequency ω and the geometrical distances d , d_1 and d_2 . Equations (13) and (14) enable us to see that the intensity of the transferred signal T is 1 and that of the reflected signal R is 0, for $k_1 d = \pi/2$. Let us define the quality factor associated with the line-width of the intensity of the transferred signal T for this peak by

$$Q(k_1 d) = \frac{k_1 d}{\Delta(k_1 d)}, \quad (19)$$

where $\Delta(k_1 d)$ is the width of this signal for $T(kd) = 0.5$.

In order to get from equation (19) an approximate value of this quality factor for the peak situated at $k_1 d = \pi/2$, one obtains first from equation (13) the value of $\Delta(k_1 d)$ for which $T = 1/2$. For $d/d_2 \gg 1$, one obtains

$$Q(k_1 d) = \frac{\pi}{4} \left(\frac{d_2}{d_1}\right)^6. \quad (20)$$

Equation (13) shows also that the intensity of the transferred signal T is 0 when $k_0 d = \arccos\left[\frac{1}{2}\left(\frac{d}{d_2}\right)^3\right]$ has real solutions. When that happens, it is also possible to find an approximative value for the quality factor of these dips. One first derives the value of $\Delta(k_1 d)$ (for which $T = 1/2$) supposed to be small compared to the value of $k_1 d$ where one has the dip. Then one uses equation (19) and obtains

$$Q(k_0 d) = 4 \left(\frac{d_1}{d}\right)^6 \left[1 - \left(\frac{d}{d_2}\right)^6\right] \arccos\left[\frac{1}{2}\left(\frac{d}{d_2}\right)^3\right]. \quad (21)$$

Let us also consider the value of the attenuation coefficient per unit length α . In order to evaluate αd we use equations (7) and (8). One may then eliminate the kd unknown by using $\sin^2(kd) + \cos^2(kd) = 1$ in order to obtain

$$\cosh(2\alpha d) = \left(\frac{\omega^2 - \omega_0^2}{2\omega_1^2}\right)^2 + \left(\frac{\Gamma_1 \omega}{2\omega_1^2}\right)^2 + \sqrt{\left(\left(\frac{\omega^2 - \omega_0^2}{2\omega_1^2}\right)^2 + \left(\frac{\Gamma_1 \omega}{2\omega_1^2}\right)^2\right)^2 - 2\left(\frac{\omega^2 - \omega_0^2}{2\omega_1^2}\right)^2 + 2\left(\frac{\Gamma_1 \omega}{2\omega_1^2}\right)^2 + 1}. \quad (22)$$

In what follows, we illustrate the above results for Ag nanoparticles having a diameter of about 20 nm. We assume [13] for Ag, $\Gamma_l = 7.9 \times 10^{13} \text{ s}^{-1}$, $\omega_0 = 5 \times 10^{15} \text{ rad s}^{-1}$ and $\omega_1 = 1.4 \times 10^{15} \text{ rad s}^{-1}$. Figures 2–4 show the transmission coefficient T as a function of the reduced frequency $(\omega/\omega_0)^2$ calculated using this point-dipole model for modes with the electric field polarized perpendicular to the chain (transverse modes).

Let us first present the case of such a system with only one adsorbed cluster. The transmitted and reflected functions can be obtained from equations (7), (13) and (14) when letting d_2 go to infinity. Without attenuation, the solid line in figure 2 gives the transmission

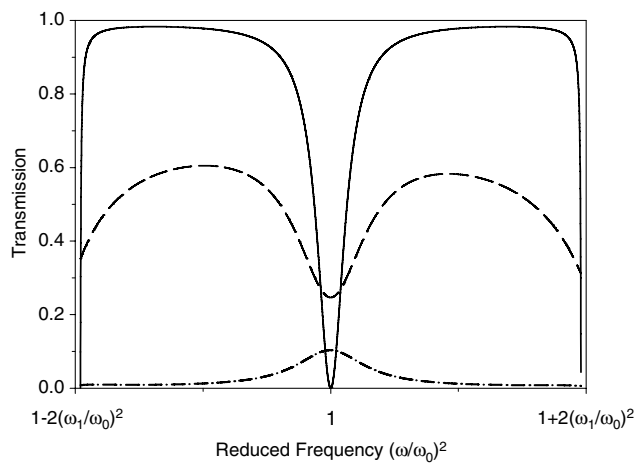


Figure 2. For one single adsorbed cluster, this figure shows the transmission coefficients T (solid line) without damping, the attenuated transmission $T \exp(-4\alpha d)$ (dashed line) and the attenuated reflectance $R \exp(-4\alpha d)$ (dotted–dashed line), as a function of the reduced frequency $(\omega/\omega_0)^2$, for $d_1/d = 1.25$.

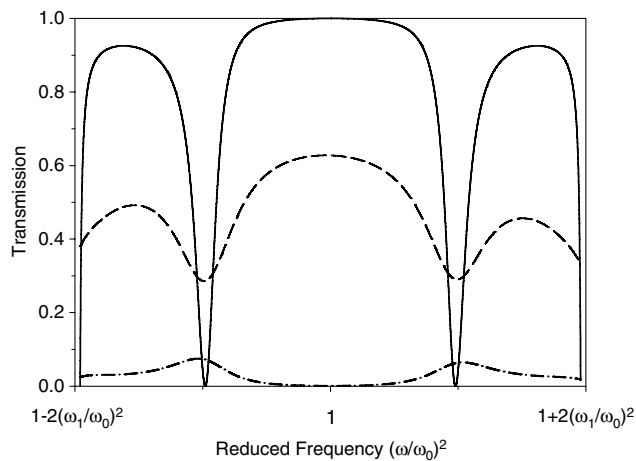


Figure 3. For two adsorbed clusters, this figure shows the transmission coefficients T (solid line) without damping, the attenuated transmission $T \exp(-4\alpha d)$ (dashed line) and the attenuated reflectance $R \exp(-4\alpha d)$ (dotted–dashed line), as a function of the reduced frequency $(\omega/\omega_0)^2$, for $d_1/d = 1.2$ and $d_2/d = 1$.

coefficient T as a function of $(\omega/\omega_0)^2$, for $d_1/d = 1.25$. The corresponding reflection coefficient (not shown) satisfies $R = 1 - T$. When attenuation is taken into account, we present in figure 2 the quantities $T \exp(-4\alpha d)$ and $R \exp(-4\alpha d)$ as dashed and dotted–dashed lines. The coefficient $\exp(-4\alpha d)$ was introduced in order to compare the transmission intensity one cluster after the resonator to the intensity one cluster before. Indeed, in some frequency domains far from ω_0 , the coefficients T or R may exceed the value of one. However, this is not contradictory with the conservation of the energy since, for instance, the power one cluster after the resonator remains always smaller than the power one cluster before the resonator. Note that,

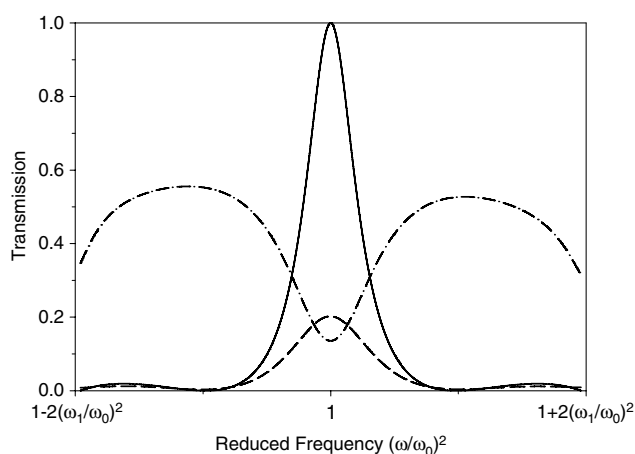


Figure 4. Same as in figure 3, but for $d_1/d = 0.7$ and $d_2/d = 1$.

without attenuation, one has a zero of transmission in the middle of the band. The quality of the corresponding dip is approximately proportional to $(d_1/d)^{12}$. It could therefore become very high. However, as one wishes this deep to be experimentally observed with the current values of the attenuation, we have to restrict ourselves to the quality shown in the present figure.

Let us now illustrate the case of the system with two adsorbed clusters. Figure 3 presents the transmission coefficients T without attenuation (solid line), $T \exp(-4\alpha d)$ with attenuation (dashed line) and $R \exp(-4\alpha d)$ with attenuation (dotted-dashed line) as a function of $(\omega/\omega_0)^2$, for $d_1/d = 1.2$ and $d_2/d = 1$. As can be seen from equation (13), the zeros of transmission were fixed by the choice of $d_2/d = 1$. The dips in the transmission coefficient without attenuation T show a width at half maximum that is of the order predicted by equation (21). The value of d_1/d could have been taken such that the dip would have very high quality; we chose the present value in order that the dips remain observable when attenuation is taken into account. The corresponding reflection curve without attenuation (not shown in this figure) is such that $R = 1 - T$. Note also that the corresponding reflection peaks with attenuation are more difficult to detect.

When decreasing d_1/d , we present figure 4 for $d_1/d = 0.7$. The peak in the transmission coefficient without attenuation T shows a width at half maximum of the order predicted by equation (20). Let us stress that the higher quality of the central peak is in agreement with the approximate relation given by equation (20). Such a system could then be used in transmission as a light filter.

The results of the present paper show that the simple structure presented in this brief report can be a filter for transverse plasmon waves. This enables us to get, in particular, a sharp peak between two zeros of transmission close to each other or sharp dips between two transmission ones. Moreover, the above-derived closed-form expressions enable us to find easily the optimal parameters for the device desired, enabling one to engineer it at will for specific applications. Let us stress that such results were not obtained [12] for a similar device with continuous wires and resonators for which only large dips were reported. Sharp peaks have been found before [12], but only for a more complicated four-lead device, namely a multiplexer.

Let us remark that, without attenuation, the zeros of transmission discussed in this brief report appear at frequencies corresponding to the eigenvalues of the isolated two-cluster molecule and the transmission one at the frequency of one single cluster. It is easy to generalize this for a bigger molecule adsorbed on one wire site. In such a case, the transmission function

is given by

$$t = \frac{1}{1 - \frac{i}{2} \frac{\omega_1^4(d_1)}{\omega_1^2(d)} \frac{g(a,a)}{\sin(ka)}}, \quad (23)$$

where $g(a, a)$ is a Green's function element of the isolated molecule; a labels the molecule cluster to which one wire cluster is bound. This shows that the zeros of transmission of such a system still appear at the eigenvalue frequencies of the isolated molecule and the transmission ones at the eigenvalue frequencies of an isolated molecule obtained from the former one without the one cluster interacting with the wire. Of course, the transmission ones appear in between the transmission zeros.

This simple nanometric filter device is expected to stimulate further research, especially for a plasmon system exhibiting much smaller damping as those considered here: like, for example, continuous metallic wires and resonant slot metallic nanoparticles [17]. In future investigations, the effects of retardation and radiation damping [18] will have to be taken into account. They will probably have important effects on the position and the widths of the sharp peaks and dips reported here. However the possibility of obtaining such structures should remain.

These results can also be generalized to other than plasmon waves. We want to stress that the very general property for such systems is that, without attenuation, a one of transmission may be adjusted to become a very high-quality peak by fitting the distances such that one mode of the isolated resonator lies between two very close modes of the resonator without the particle interacting with the one-dimensional waveguide.

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